MEI

MEI STRUCTURED MATHEMATICS

INTRODUCTION TO ADVANCED MATHEMATICS, C1

Practice Paper C1-B

Additional materials: Answer booklet/paper
Graph paper
MEI Examination formulae and tables (MF12)

TIME 1 hour 30 minutes

INSTRUCTIONS

- Write your Name on each sheet of paper used or the front of the booklet used.
- Answer **all** the questions.
- You **may** use a graphical calculator in this paper.

INFORMATION

- The number of marks is given in brackets [] at the end of each question or part-question.
- You are advised that you may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is **72**.

Section A (36 marks)

1 Make a the subject of the equation $s = ut + \dfrac{1}{2}at^2$. [3]

2 **(i)** Find the constants a and b such that, for all values of x,

$$x^2 + 4x + 14 = (x + a)^2 + b.$$ [3]

  (ii) Write down the greatest value of $\dfrac{1}{x^2 + 4x + 14}$. [1]

3 Find the term independent of x in the expansion of $\left(x - \dfrac{2}{x} \right)^4$. [3]

4 The coordinates of the points A, B and C are (-2, 2), (1, 3) and (3, -3) respectively.

  (i) Find the gradients of the lines AB and BC. [2]

  (ii) Show that the triangle ABC is a right-angled triangle. [1]

  (iii) Find the area of the triangle ABC. [2]

5 You are given that $f(x) = x^3 - 7x + 6$.

  (i) Show that $(x - 2)$ is a factor of $f(x)$. [1]

  (ii) Solve the equation $f(x) = 0$. [4]

6 List the integers which satisfy both of the following inequalities:

$$2x - 9 < 0, \quad 8 - x \le 6.$$ [3]

7 **(a)** Express $\left(2 + \sqrt{3} \right)^2$ in the form $a + b\sqrt{3}$ where a and b are integers to be determined. [2]

  (b) Given that x and y are integers, prove that $\dfrac{1}{x - \sqrt{y}} + \dfrac{1}{x + \sqrt{y}}$ can be written in the

  form $\dfrac{p}{q}$ where p and q are both integers. [3]

8 Find the equation of the line that passes through the point (1, 2) and is perpendicular to the line $3x + 2y = 5$. [3]

9 **(i)** Show that $(x - 1)(x - 2)(x - 3) - (x^3 - x^2 + 11x - 12) = 6 - 5x^2$. [3]

(ii) Solve the equation $6 - 5x^2 = 0$. [2]

Section B (36 marks)

10 **(i)** A quadratic function is given by $f(x) = x^2 - 6x + 8$.
Sketch the graph of $y = f(x)$, giving the coordinates of the points where it crosses the axes. Mark the lowest point on the curve, and give its coordinates. [5]

(ii) Solve the inequality $x^2 - 6x + 8 < 0$. [2]

(iii) On the same graph, sketch $y = f(x + 3)$. [2]

(iv) The graph of $y = f(x + 3) - 2$ is obtained from the graph of $y = f(x)$ by a transformation. Describe the transformation and sketch the curve on the same axes as in **(i)** and **(iii)** above. Label all these curves clearly. [3]

11 **(i)** Show algebraically that the equation $x^2 - 6x + 10 = 0$ has no real roots. [2]

(ii) Solve algebraically the simultaneous equations $y = x^2 - 6x + 10$ and $y + 2x = 7$. [4]

(iii) Plot the graph of the function $y = x^2 - 6x + 10$ on graph paper, taking 1 cm = 1 unit on each axis, with the x axis from 0 to 6 and the y axis from -2 to 10.
On the same axes plot the line with equation $y + 2x = 7$ showing clearly where the line cuts the quadratic curve. [3]

(iv) Explain why these x coordinates satisfy the equation $x^2 - 4x + 3 = 0$.
Plot a graph of the function $y = x^2 - 4x + 3$ on the same axes to illustrate your answer. [3]

12 You are given that the equation of the circle shown in Fig. 12 is
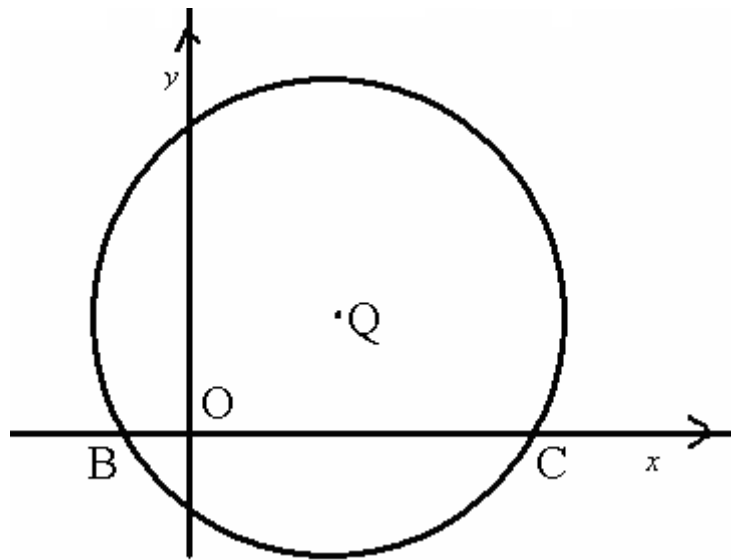
$$x^2 + y^2 - 4x - 6y - 12 = 0$$

Fig. 12

(i) Show that the centre, Q, of the circle is (2, 3) and find the radius. [4]

(ii) The circle crosses the *x*-axis at B and C.
Show that the coordinates of C are (6, 0) and find the coordinates of B. [3]

(iii) Find the gradient of the line QC and hence find the equation of the tangent to the circle at C. [3]

(iv) Given that M is the mid-point of BC, find the coordinates of the point where QM meets the tangent at C. [2]